VOL. 84, NO. A6 JOURNAL OF GEOPHYSICAL RESEARCH JUNE 1, 1979

Conductorlike Behavior of a Photoemitting Dielectric Surface

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We provide an analytical proof showing that a uniformly illuminated photoemitting dielectric surface where the photoelectron emission has an azimuthal symmetry becomes an equipotential surface in the steady state. This result supports the finding of an earlier numerical simulation study of the problem.

It has been suggested in the past that a uniformly illuminated photoemitting dielectric surface of finite extent acquires in the steady state a surface charge distribution as if the surface were conducting, i.e., the surface becomes equipotential [*De and Criswell*, 1977*a*, *b*; *Criswell and De*, 1977]. This suggestion has important implications to the problem of electrical charging of spacecrafts (see, e.g., *Pike and Lovell* [1977]) and possibly also to the problem of localized charging of the lunar surface [*Criswell*, 1973]. The theoretical suggestion was based on heuristic and plausibility arguments and was later substantiated by detailed numerical simulation studies [*Pelizzari and Criswell*, 1978]. In this paper we provide an analytical proof of the conductorlike behavior. The only restrictions we place on the problem are that the photoelectron emission from the surface have an azimuthal symmetry and that the photosheath may be assumed to be collisionless. We tacitly assume that a steady state is attainable, which means that the photoelectron spectrum has a high-energy cutoff.

In the steady state, let $f(u, v, w)$ be the velocity distribution function of the electrons, where u, v, and w are the x, y, and z components of the photoelectron velocity $\mathbf{V}$. The x and y axes are taken to lie on the dielectric surface, and the z axis is perpendicular to this surface. Since in the steady state the net charge of any volume element in the photosheath remains unchanged with time, we have

$$\frac{1}{e}\nabla\cdot\mathbf{J} = \nabla\cdot\left(\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f\mathbf{V}\,du\,dv\,dw\right) = 0 \tag{1}$$

where $\mathbf{J}$ is the electric current and e is the electronic charge. Since for $V = 0$, $f = 0$, and since $V^2 = u^2 + v^2 + w^2$, it follows that the function f is continuous at $u = 0$, $v = 0$, or $w = 0$. Furthermore, since the distribution function f must vanish whenever any of the velocity components becomes infinite, we can rewrite the above equation (without any loss of generality) as

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathbf{V}\cdot\nabla f\,du\,dv\,dw = 0 \tag{2}$$

where use has been made of the fact that $\nabla\cdot\mathbf{V} = 0$.

Using the collisionless Boltzmann equation in the steady state, we find further that

$$\mathbf{V}\cdot\nabla f = -\frac{e}{m}\nabla\phi\cdot\frac{\partial f}{\partial\mathbf{V}} \tag{3}$$

where ϕ is the electric potential at any point in the sheath (including points on the surface) and m is the electron mass. Combining (2) and (3), we find that at any point in the sheath,

$$\left(\frac{\partial\phi}{\partial x}\right)\iiint\left(\frac{\partial f}{\partial u}\right)du\,dv\,dw + \left(\frac{\partial\phi}{\partial y}\right)\iiint\cdot\left(\frac{\partial f}{\partial v}\right)du\,dv\,dw + \left(\frac{\partial\phi}{\partial z}\right)\iiint\left(\frac{\partial f}{\partial w}\right)du\,dv\,dw = 0 \tag{4}$$

where the limits of the integrals are the same as those in (2) and shall be understood in the following discussion. Note now that in the steady state the net flux of electrons in the z direction anywhere on the surface must vanish, i.e.,

$$\iiint fw\,du\,dv\,dw = 0 \tag{5}$$

which indicates that f is an even function of w. It therefore follows that the last integral in (4) vanishes at the surface. This leaves us with a condition that must hold at any point on the surface:

$$E_x\iiint\left(\frac{\partial f}{\partial u}\right)du\,dv\,dw + E_y\iiint\left(\frac{\partial f}{\partial v}\right)du\,dv\,dw = 0 \tag{6}$$

where E_x and E_y are the x and y components of the electric field at the surface.

We shall next divide f into two components: $f = f^{\uparrow} + f^{\downarrow}$, where $f^{\uparrow}$ refers to electrons with $w > 0$ and $f^{\downarrow}$ refers to electrons with $w < 0$. Because of the uniform illumination of the surface, $f^{\uparrow}$ has the same value at all points on the surface. Because of this fact and the assumed azimuthal symmetry of electron emission, there is no net drift of the electrons with $w > 0$ parallel to the surface at the surface, so that

$$\iiint f^{\uparrow}u\,du\,dv\,dw = 0 \tag{7}$$

$$\iiint f^{\uparrow}v\,du\,dv\,dw = 0 \tag{8}$$

This means that $f^{\uparrow}$ is an even function of u and v, so that the portions of the integrals in (6) for $w > 0$ vanish, leaving

$$E_x\iiint\left(\frac{\partial f^{\downarrow}}{\partial u}\right)du\,dv\,dw + E_y\iiint\left(\frac{\partial f^{\downarrow}}{\partial v}\right)du\,dv\,dw = 0 \tag{9}$$

as the steady state condition applicable at the surface.

Suppose that at a certain point on the surface there is a nonzero electric field component parallel to the surface. We can choose our coordinate system in such a way that $E_x = 0$ and $E_y \neq 0$. This reduces condition (9) to

$$E_y\iiint\left(\frac{\partial f^{\downarrow}}{\partial v}\right)du\,dv\,dw = 0 \tag{10}$$

A nonzero E_y requires that $f^{\downarrow}$ incorporate a net drift of the electrons in the y direction, i.e., that $f^{\downarrow}$ be in part an odd

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Paper number 8A1259.
0148-0227/79/008A-1259$01.00

function of v. This makes the above integral nonzero, so that we are faced with an absurdity. The direct conclusion is that there can be no electric field component parallel to the surface anywhere on the surface. The surface must consequently be equipotential.

If we give up the assumption of uniform illumination or azimuthal symmetry, then the electrons with $w > 0$ can have an inherent drift along the surface, so that the preceding discussion does not apply. In this case the surface need not be an equipotential, although the basis condition expressed by (6) must apply.

Acknowledgments. I thank D. R. Criswell and M. A. Pelizzari for helpful discussions. The Lunar and Planetary Institute is operated by the Universities Space Research Association under contract NSR 09-051-001 with the National Aeronautics and Space Administration. This work constitutes Lunar and Planetary Institute contribution 323.

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(Received April 25, 1978;
accepted November 16, 1978.)